Impacts of presentation of higher level mathematics topics to freshman students with hands-on activities with an emphasis on topology

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Focus of the study

- Natural Science students
- First Year Seminar – The art of thinking: Big picture of mathematics
- Motivation of differentiation:
  - To what extent participants are aware of creativity in math
  - To what extent mathematical creativity can be improved.
- With this study:
  - challenge students' perception towards mathematics
  - observe a possible change of this perception and its impact on motivation towards mathematics

Math Creativity

- Lack of recorded creative thinking procedure of mathematicians
  - drill + practice
  - deductive thinking

Conclusion: Hindered creative nature of math

- Need for a change of mindset
- Combination of intuitional and formalized thinking
- ”Creativity plays a vital role in the advanced mathematics” (Ervynck, 1991).

Characteristic of Creative Math	Mathematical Creativity Skills	Observed through explanation or example of (increased) common themes of these characteristics and skills
Math is relational (Ervynck, 1991): It stimulates through interaction: It establishes a conceptual link between two or more concepts	Relating previously unrelated concepts	**Making associations**
Is deep - needs uncovering the hidden relationships (Polya, 1945, 1954).	Reformulating problems or finding analogous problems	**Analogical thinking**
Has interdisciplinary and emergent applications (Steen, 2001): It integrates different aspects from different disciplines to model what is happening in the real world	"Mathematizing" reality	**Interdisciplinary thinking**

Characteristic of Creative Math	Mathematical Creativity Skills	Observed through explanation or example of (increased) common themes of these characteristics and skills
Has mathematical mutation	Knowing the qualifying criterion for bein valuable in math and Selection of the m fitted mathematical concepts/theorems/axioms/solutions bas on these criteria	**Selective thinking**
Strives economy of thought (Krutetskii, 1969)	Finding the most economical aka clear, simple, and yet 'elegant' ways to solve problems	**Appreciation of simplicity/aesthetics**
Condensing and flexible (Krutetskii, 1969)	Ability to choose appropriate wording ar symbols for the representation of mathematical concepts and being able easily switch from one representation t another.	**Use of different representations**

Characteristic of Creative Math	Mathematical Creativity Skills	Observed through explanation or example of (increased) common themes of these characteristics and skills
Original (Sriraman, 2005)	a) Flexibility of thought and the breakin from mental sets b) Move towards a formulation of new knowledge-reorientation to consolidate what is important, envision of what will b important in the future, rearrangement c known results	**Unique - Unorthodox problem solving**
Needs cognizance of the thinker's thought process (Carlson & Bloom, 2005)	Developing creative metacognition "a combination of creative self-knowledge (knowing one's own creative strengths and limitations, both within a domain ar as a general trait) and contextual knowledge (knowing when, where, hov and why to be creative)"	**Metacognitive thinking**

Math Creativity

- Literature shows evidence of a significant relationship between motivation toward mathematics and mathematical creativity.
- The more positive attitude towards math, the greater the likelihood of indications/indicators of mathematical creative
- Understanding that mathematical thinking is a creative process helps students to reveal their creative potential in math

Math Education

- Need for advanced mathematical techniques - content to show mathematical creativity (Cropley 1999)
- Encouragement of integration of hands-on activities and helping students for incremental improvements (Sheffield 2009)
- There are only a few of studies conducted supporting the aforementioned key points, and most of them with K-12 Pupils

Math Education

- For higher education (Robert and Schwarzenberger 2002):
  - Content presentation (lecture + homework based)
  - More inaccessible content leading unimaginative teaching techniques

Advanced and creative content +
Differentiated instruction method

= PROMOTION OF CREATIVITY!

Class Design

- First-Year Seminar at an R1 state university named "The Art of Thinking: Big Picture of Mathematics"
- Math games / activities in groups of 3-4 people each week
- Strong connection between the activities and the theoretic and abstract facts mathematics
- Various topics in math such as combinatorics, set theory, number theory and topology

Class Design

- Video recordings containing an explanation of the bridge between that week's activity and its related theory
- Reflective journals about how they feel after learning about the theory

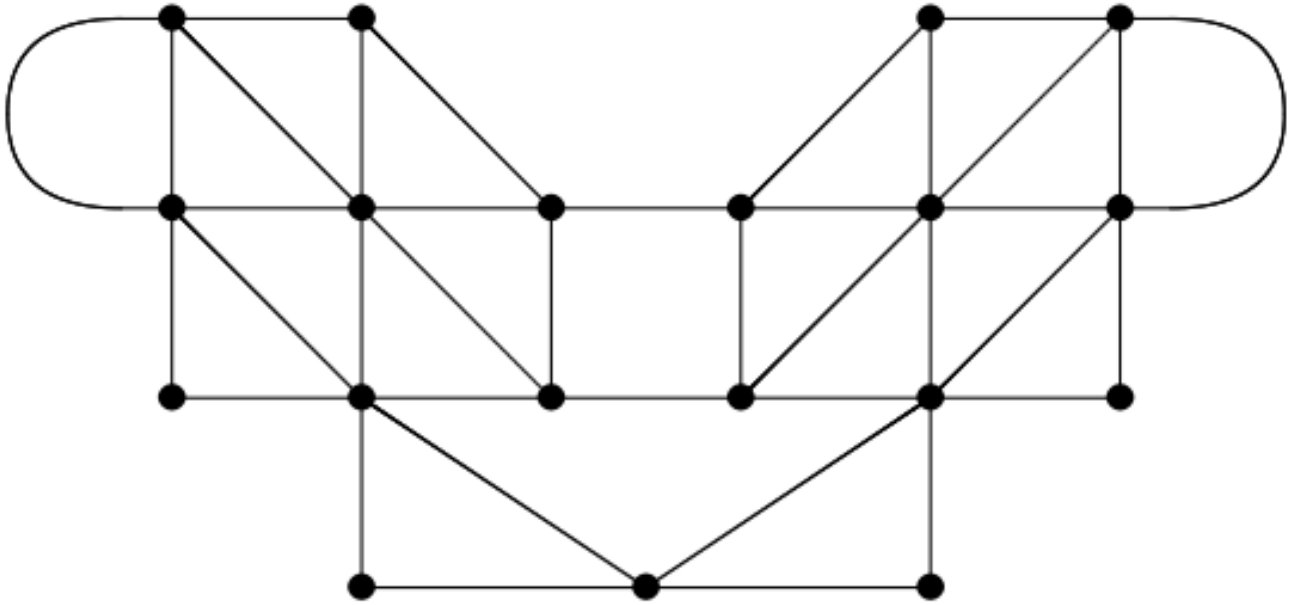
Math Education Goal of this intervention

To provide an environment that stimulates creativity and enables thinking creatively and introspectively in mathematical thinking in higher education.

Math Education

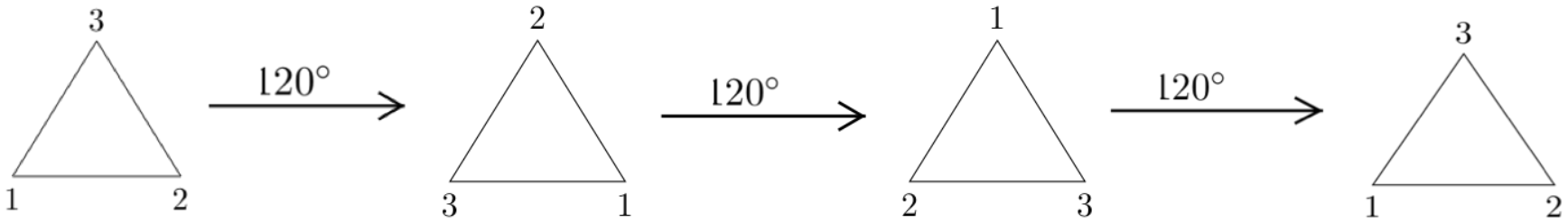
- How to fulfill this goal:
  - More hands-on approach - not directly lecture based
  - Advanced level theoretic / abstract topics turned into games / activities
  - Presenting the theory behind explicitly after the games / activities to allow the incubation period

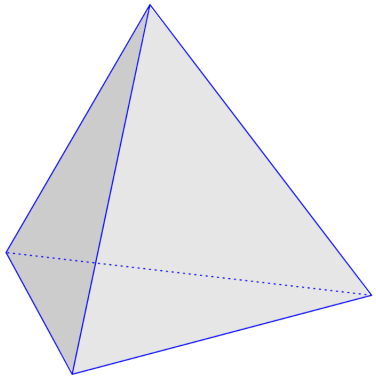
Activity - Bridges of Konigsberg



Theme: Graph Theory - Topology

Activity - Symmetries of Shapes





Theme: Group Theory - Topology

Activity - Groups and Fields

Let us recall the definition of a group. A group $(G, *)$ is a set G with some operation $*$ that satisfies the following three properties:

(a) (Closedness) For every two element a, b of G, $a * b$ also is an element of G.

(b) (Identity Element) There is an identity element e of G. In other words, for any element a of G, $a * e = e * a = a$.

(c) (Invertibility) For any element a of G, there is an inverse $\tilde{a}$. In other words, there is an element $\tilde{a}$ such that $a * \tilde{a} = \tilde{a} * a = e$.

Question 1: Can you give a full list of elements of $\mathbb{Z}/4\mathbb{Z}$?

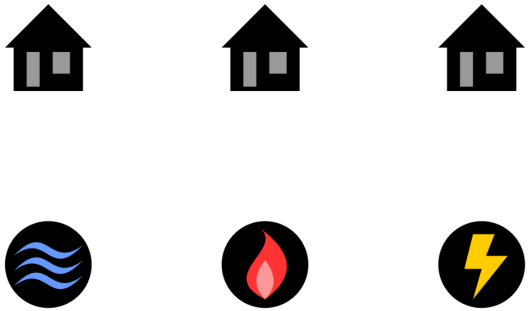
Question 2: Can you give a full list of elements of $\mathbb{Z}/n\mathbb{Z}$ for any positive integer n?

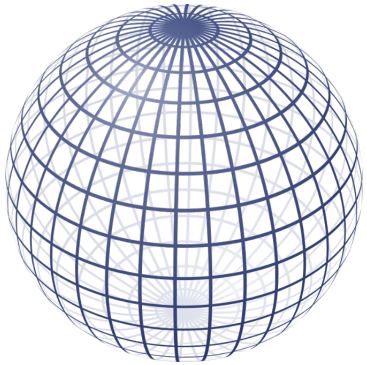
Question 3: Let us try to fill out the following table. By looking at the values, can you verify that $\mathbb{Z}/4\mathbb{Z}$ is a group under addition? In other words, does this set satisfy the 'group properties'?

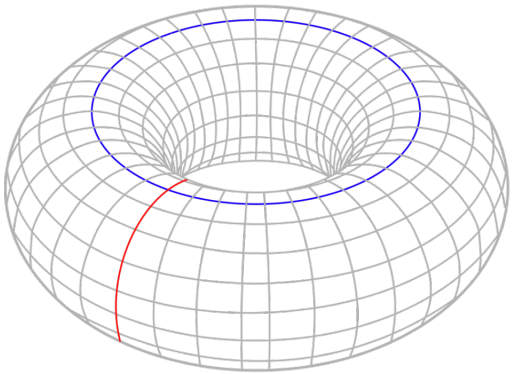
$+$	0	1	2	3
0				
1				
2				
3				

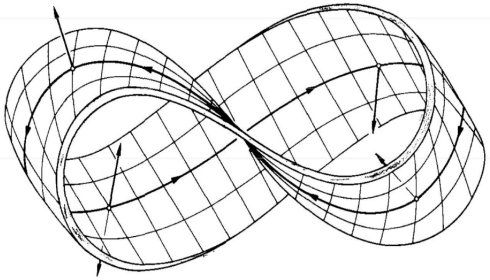
Theme: Group Theory - Modular Arithmetic

Activity - Three Utilities Problem









Theme: Topology

Purpose of the study

- To examine the relationship between motivation toward mathematics and math content
- To examine the relationship between math creativity and math content

Research Questions

Quantitative:

- To what extent does a creativity-based math content change the motivation towards math?

Qualitative:

- What changes are observed in students' perception of math with the use of creativity-based math content?
- What changes are observed in students' motivation towards math with the use of creativity-based math content?
  - perception: gaining insights about creativity in math

Instruments - Quantitative

- *How I Feel About Math* (Mann, 2005)
  - students' attitude towards math
  - to explore the relationship between differentiated *content* and *attitude* towards mathematics.
- *The Effectance Motivation Scale*
  - student's motivation for involvement in mathematics.
  - cronbach's alpha = .86

Effectance questions: 2,5,8,9,13,21,22,28,37,39,40

- Non-parametric Wilcoxon Signed Rank Test to compare pre-post test scores

Results

Table 1. Wilcoxon Signed Ranks Test Results Between Pre-Test and Post-Test Results on the Effectance Motivation Subscale

Measurement of pre-post test	N	Mean Rank	Sum of Ranks	z	p
Negative Ranks	2	4.25	8.50	-1.948	.05
Positive Ranks	8	5.81	46.50		
Ties	1				

Instruments - Qualitative

- Weekly reflective journals
  - thoughts
  - solutions
  - related ideas
  - feelings

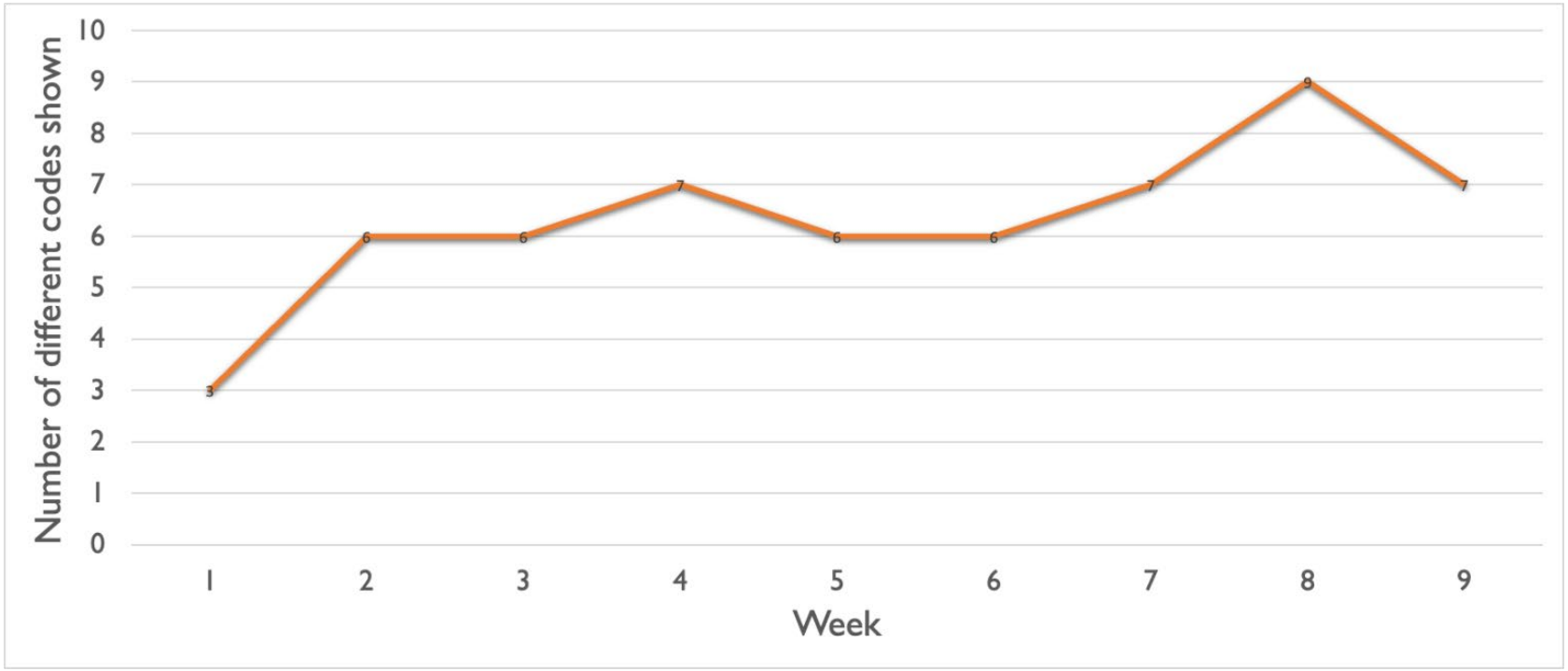
Data Analysis and Interpretation - Qualitative

- Emerging Themes in reflective journals
- 2 main thematic categories
  - motivation
    - The Effectance Motivation Scale
      - 3 codes detected - which might be related to the significant increase in the subscale scores.
      - Motivation for further study
  - math creativity

Data Analysis and Interpretation - Qualitative

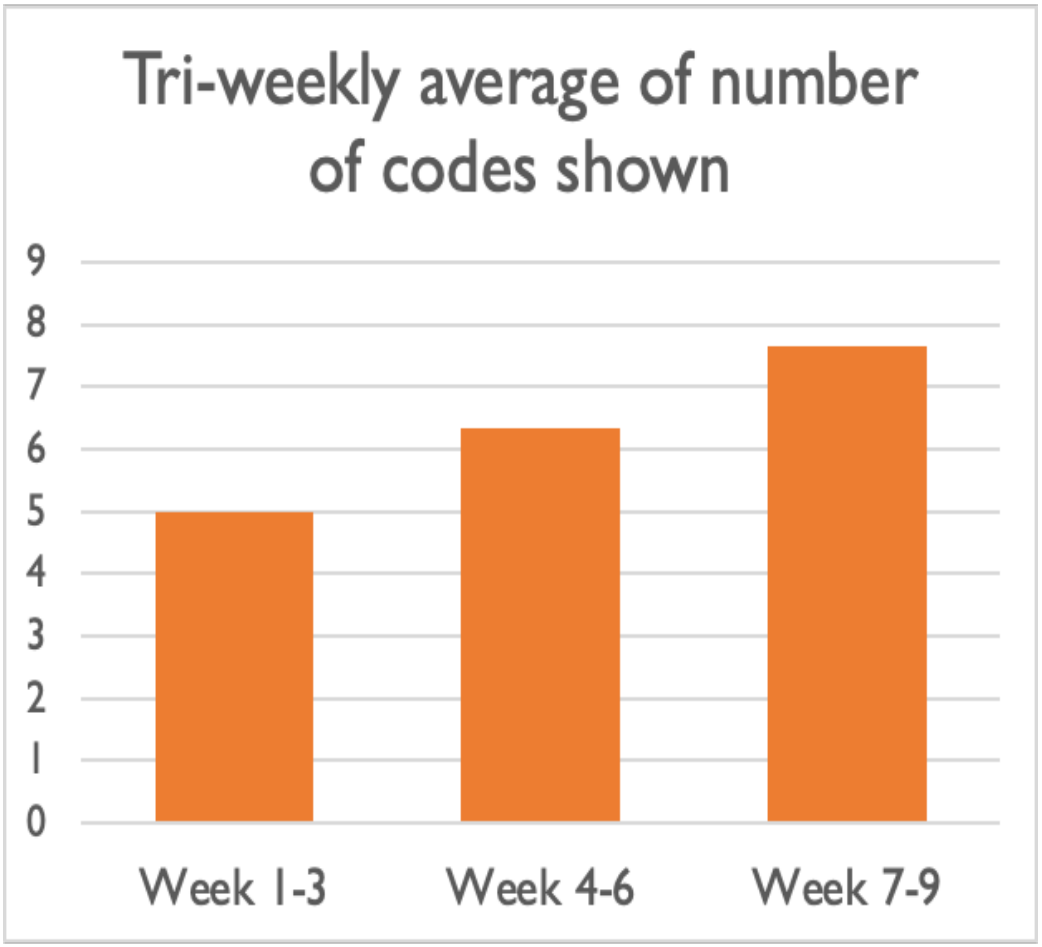
- Increase in:
  - the frequency - occurrence of mathematical creativity-related codes
  - the frequency of mathematical motivation codes
  - the number of students showing those codes
  - the number of different codes shown by week

Data Analysis and Interpretation - Qualitative



- Increase in the number of codes shown by week
- Increasing trend, but slight decreases between a few weeks
- They can be related to the factors that are not controlled in this study, such as the readiness of the student to the content of that week, and the timing of the meeting (exam week, etc.)

Data Analysis and Interpretation - Qualitative



- In order to normalize this effect, we check the average number of skills for three-week periods

Data Analysis and Interpretation - Qualitative - Motivational Codes

Motivational Code	Number of students			Number of Occurrances			Examplery Quote
	W 1-3	W 4-6	W 7-9	W 1-3	W 4-6	W 7-9	
Having fun - emotions	5	7	6	8	9	11	But the axiom of choice implies that you can enumerate this set, which led to the strange result that a sphere could be decomposed into two identical versions of itself.
Curiosity - asking meaningful questions	3	2	4	4	4	6	So why do we have the p-adic numbers? What problem do they solve? I believe they solve two problems…
Meta-math / questioning the background	3	4	5	4	6	9	Rethinking things like distance, which we usually take for granted, led to a lot of interesting consequences and new questions.

- Consistent increase in the number of occurrences
- Overall increase in the number of students but not consistent
- Some students showing more improvement
- Several reasons: readiness of the student, the relationship between the student and the instructor and the student's relationship within the group

Data Analysis and Interpretation - Qualitative - Mathematical Creativity Codes

Math Creativity Code	Number of students			Number of Occurrances			Examplery Quote
	W 1-3	W 4-6	W 7-9	W 1-3	W 4-6	W 7-9	
Use of metaphor-anologies	0	1	2	0	2	4	I usually think of modulo like a clock or circle
Interdisciplinary thinking	1	1	4	1	2	5	The description of how groups can be used in cryptography reminds me of another example I've seen that used rational numbers.
Making associtations	3	4	5	4	6	7	For the Königsberg bridge problem it was parity associated with odd nodes, for the Hilbert hotels it was injective functions, and for symmetries it was the recognition of a group structure.
Selective thinking	2	3	4	2	4	4	The first exercises were easy because they all relied upon the axiom of choice and countably infinite sets. The third exercise was tricky because…
Unique - Unorthodox problem solving	1	2	4	1	2	6	This led to new notation and tied into our activity from last week with equilateral triangles.

Data Analysis and Interpretation - Qualitative - Mathematical Creativity Codes

Math Creativity Code	Number of students			Number of Occurrances			Examplery Quote
	W 1-3	W 4-6	W 7-9	W 1-3	W 4-6	W 7-9	
Metacognitive thinking	2	2	2	2	2	3	This is an easy concept to understand, yet I found myself struggling with some of the problems.
Appreciation of simplicity/aesthetics	2	3	5	3	3	6	If there's one thing that I have learned from this week's exercise, it is that the best discoveries come from simple ideas.
Use of different representations	1	1	2	1	2	2	The idea that powers Cantor's infinities is injectivity, as seen in the picture below, the function f: A → B maps every element of A onto a unique element of B.

Main Take-Away

- Increased motivation towards math enables to observe indicators of mathematical creative
- Understanding and experiencing that mathematical thinking is a creative process help students to reveal their creative potential in math

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Thanks!

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